

# Normal solution to the Enskog-Landau kinetic equation. Boundary conditions method

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## Abstract

Nonstationary and nonequilibrium processes are considered on the basis of an Enskog-Landau kinetic equation using a boundary conditions method. A nonstationary solution of this equation is found in the pair collision approximation. This solution takes into account explicitly the influence of long-range interactions. New terms to the transport coefficients are identified. An application of the boundary conditions method to hydrodynamic description of fast processes is discussed.

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The development of methods to construct a theory for nonequilibrium processes in dense gases, liquids and plasmas is an important direction in the modern theoretical physics. Moreover, the construction of kinetic equations for such classical and quantum systems still remains to be a major problem in the kinetic theory. It is complicated additionally in the case of dense gases, liquids and plasmas, where kinetics and hydrodynamics are closely connected and should be considered simultaneously [1–5].

An approach for construction of kinetic equations from the first principles of statistical mechanics, namely from the Liouville equation, has been developed in [6,7]. Another approach for obtaining kinetic equations for *dense* systems, which is based on ideas of papers [6,7], has also been proposed [1] and generalized in [2,3]. Here, the formulation of modified boundary conditions for the BBGKY hierarchy is used taking into account corrections connected with local conservation laws. On the basis of this sequential approach, a new Enskog-Landau kinetic equation has been obtained for an one-component system of charged hard spheres. There is a considerable interest of an application of this kinetic equation for description of transport processes in dense systems of

charged particles as well as in ion melts and electrolyte solutions. The normal solution and transport coefficients for this equation have been found in the paper [8] using the Chapman-Enskog method. The same approach has been used for a many-component system of charged hard spheres in the paper [9] with more detailed calculations for a two-component system as well. At the same time, as is well known, the Chapman-Enskog method allows one to find the transport coefficients in a stationary case only. Similar drawbacks are peculiar to the Grad method [10,11] which is often used to solve kinetic equations next to the Chapman-Enskog method.

In this paper, the Enskog-Landau kinetic equation for a system of charged hard spheres is investigated. To find the normal solution in a nonstationary case, the so-called boundary conditions method is used, which has been introduced in [12,13]. As a result, transport coefficients equations in the nonstationary case are written. A limiting case of the stationary process is considered. A brief comparison of the obtained transport coefficients with those known previously from the Chapman-Enskog method is given.

Let us consider the Enskog-Landau kinetic equation for a one-component system of charged hard spheres [8]:

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v}_1 \frac{\partial}{\partial \mathbf{r}_1} \right\} f_1(\mathbf{x}_1; t) = I_E(\mathbf{x}_1; t) + I_{MF}(\mathbf{x}_1; t) + I_L(\mathbf{x}_1; t), \quad (1)$$

where  $f_1(\mathbf{x}_1; t)$  is the one-particle distribution function. The right-hand side of this equation is the so-called generalized Enskog-Landau collision integral, where each term can be considered as a separate collision integral. Their structure are as follows:

$I_E(\mathbf{x}_1; t)$  is the collision integral of the Enskog theory RET [14]:

$$\begin{aligned} I_E(\mathbf{x}_1; t) = & \sigma^2 \int d\hat{\mathbf{r}}_{12} d\mathbf{v}_2 \Theta(\hat{\mathbf{r}}_{12}\mathbf{g})(\hat{\mathbf{r}}_{12}\mathbf{g}) \times \\ & \left\{ g_2(\mathbf{r}_1, \mathbf{r}_1 + \hat{\mathbf{r}}_{12}\sigma; t) f_1(\mathbf{r}_1, \mathbf{v}'_1; t) f_1(\mathbf{r}_1 + \hat{\mathbf{r}}_{12}\sigma, \mathbf{v}'_2; t) - \right. \\ & \left. g_2(\mathbf{r}_1, \mathbf{r}_1 - \hat{\mathbf{r}}_{12}\sigma; t) f_1(\mathbf{r}_1, \mathbf{v}_1; t) f_1(\mathbf{r}_1 - \hat{\mathbf{r}}_{12}\sigma, \mathbf{v}_2; t) \right\}, \end{aligned} \quad (2)$$

where  $\sigma$  is a hard sphere diameter,  $\mathbf{g}$  denotes a vector of relative velocity for two particles,  $\hat{\mathbf{r}}_{12}$  is a unit vector along the direction between centres of particles 1 and 2,

$$\begin{aligned} \mathbf{v}'_1 &= \mathbf{v}_1 + \hat{\mathbf{r}}_{12}(\hat{\mathbf{r}}_{12} \cdot \mathbf{g}), & \mathbf{g} &= \mathbf{v}_2 - \mathbf{v}_1, \\ \mathbf{v}'_2 &= \mathbf{v}_2 - \hat{\mathbf{r}}_{12}(\hat{\mathbf{r}}_{12} \cdot \mathbf{g}), & \hat{\mathbf{r}}_{12} &= |\mathbf{r}_{12}|^{-1} \mathbf{r}_{12}; \end{aligned}$$

$I_{MF}(x_1; t)$  is the collision integral of the kinetic mean field theory KMFT [15,16]:

$$I_{MF}(x_1; t) = \frac{1}{m} \int dx_2 \frac{\partial \Phi^l(|\mathbf{r}_{12}|)}{\partial \mathbf{r}_1} \frac{\partial}{\partial \mathbf{v}_1} g_2(\mathbf{r}_1, \mathbf{r}_2; t) f_1(x_1; t) f_1(x_2; t), \quad (3)$$

where  $\Phi^l(|\mathbf{r}_{12}|)$  is a long-range part of the interparticle interaction potential;

$I_L(x_1; t)$  is generalized Landau collision integral [2,8]:

$$I_L(x_1; t) = \frac{1}{m^2} \frac{\partial}{\partial \mathbf{v}_1} \int dx_2 g_2(\mathbf{r}_1, \mathbf{r}_2; t) \left[ \frac{\partial \Phi^l(|\mathbf{r}_{12}|)}{\partial \mathbf{r}_{12}} \right] \times \quad (4)$$

$$\left[ \int_{-\infty}^0 dt' \frac{\partial \Phi^l(|\mathbf{r}_{12} + \mathbf{g}t'|)}{\partial \mathbf{r}_{12}} \right] \left\{ \frac{\partial}{\partial \mathbf{v}_1} - \frac{\partial}{\partial \mathbf{v}_2} \right\} f_1(x_1; t) f_1(x_2; t).$$

It is necessary to note that the quasiequilibrium binary correlation function  $g_2$  takes into account the full interaction potential (hard core part plus long-range Coulomb tail).

One of a major problem at the correct derivation and solution of kinetic equations is their consistency with local conservation laws of particle density (or mass), momentum, total energy and substantiation of hydrodynamic equations and incomprehensible calculation of transport coefficients via molecular parameters. These conservation laws for classical systems in general have the structure as in [17].

To find a solution of the Enskog-Landau kinetic equation (1) using one or another method, there is necessary to take the advantage of local conservation laws in corresponding approximations. So doing the expressions for kinetic coefficients will be defined through densities for momentum flow tensor  $\vec{\Pi}(\mathbf{r}; t)$  and energy flow vector  $\mathbf{j}_\mathcal{E}(\mathbf{r}; t)$  on the basis of solution  $f_1(x_1; t)$  and corresponding approximations for  $g_2(\mathbf{r}_1, \mathbf{r}_2; t)$ . As far as we find the solution that corresponds to linear hydrodynamical transport processes by gradients of thermodynamical parameters, densities of momentum flow tensor  $\vec{\Pi}(\mathbf{r}; t)$  and energy flow vector  $\mathbf{j}_\mathcal{E}(\mathbf{r}; t)$  could be determined immediately with the help of kinetic equation (1) without general formulas from [17]. To this end it is convenient similarly to [2], to introduce the following hydrodynamical parameters: density  $n(\mathbf{r}_1; t)$  (or mass density  $\rho(\mathbf{r}_1; t)$ ), hydrodynamical velocity  $\mathbf{V}(\mathbf{r}_1; t)$  and density of kinetic energy  $\omega_k(\mathbf{r}_1; t)$ . Multiplying initial kinetic equation (1) by hydrodynamical parameters and integrating with respect to  $\mathbf{v}_1$ , one can obtain the equations for these parameters in the form:

$$\frac{1}{\rho(\mathbf{r}_1; t)} \frac{d}{dt} \rho(\mathbf{r}_1; t) = - \frac{\partial}{\partial \mathbf{r}_1} \mathbf{V}(\mathbf{r}_1; t), \quad (5)$$

$$\rho(\mathbf{r}_1; t) \frac{d}{dt} \mathbf{V}(\mathbf{r}_1; t) = - \frac{\partial}{\partial \mathbf{r}_1} : \overleftrightarrow{P}(\mathbf{r}_1; t), \quad (6)$$

$$\rho(\mathbf{r}_1; t) \frac{d}{dt} w_k(\mathbf{r}_1; t) = - \frac{\partial}{\partial \mathbf{r}_1} \mathbf{q}(\mathbf{r}_1; t) - \overleftrightarrow{P}(\mathbf{r}_1; t) : \frac{\partial}{\partial \mathbf{r}_1} \mathbf{V}(\mathbf{r}_1; t), \quad (7)$$

where

$$\begin{aligned} \overleftrightarrow{P}(\mathbf{r}_1; t) &= \overleftrightarrow{P}^{\leftarrow k}(\mathbf{r}_1; t) + \overleftrightarrow{P}^{\leftarrow hs}(\mathbf{r}_1; t) + \overleftrightarrow{P}^{\leftarrow mf}(\mathbf{r}_1; t) + \overleftrightarrow{P}^{\leftarrow l}(\mathbf{r}_1; t), \\ \mathbf{q}(\mathbf{r}_1; t) &= \mathbf{q}^k(\mathbf{r}_1; t) + \mathbf{q}^{hs}(\mathbf{r}_1; t) + \mathbf{q}^{mf}(\mathbf{r}_1; t) + \mathbf{q}^l(\mathbf{r}_1; t) \end{aligned} \quad (8)$$

are the total stress tensor and vector of heat flow correspondingly. They have additive structure and contain several terms, each of them is stipulated by the influence from one of collision integrals [2,8]:  $\overleftrightarrow{P}^{\leftarrow hs}$  and  $\mathbf{q}^{hs}$  by Enskog collision integral (2),  $\overleftrightarrow{P}^{\leftarrow mf}$  and  $\mathbf{q}^{mf}$  by collision integral of KMFT (3),  $\overleftrightarrow{P}^{\leftarrow l}$  and  $\mathbf{q}^l$  by Landau collision integral (4),  $\overleftrightarrow{P}^{\leftarrow k}$  and  $\mathbf{q}^k$  are pure kinetic contributions only.  $\overleftrightarrow{P}^{\leftarrow l}(\mathbf{r}_1; t)$  and  $\mathbf{q}^l(\mathbf{r}_1; t)$  are new terms in the structure of (8) in comparison with results of [18]:

$$\begin{aligned} \overleftrightarrow{P}^{\leftarrow l}(\mathbf{r}_1; t) &= \\ \frac{Z^4 e^4}{m} \int d\mathbf{v}_1 \mathbf{v}_1 \frac{\partial}{\partial \mathbf{v}_1} \int dx_2 \frac{\mathbf{r}_{12} \mathbf{r}_{12}}{r_{12}^5} \cdot \frac{\mathbf{r}_{12}}{g} \left\{ \frac{\partial}{\partial \mathbf{v}_1} - \frac{\partial}{\partial \mathbf{v}_2} \right\} \int_0^1 d\lambda F^l, \end{aligned} \quad (9)$$

$$\begin{aligned} \mathbf{q}^l(\mathbf{r}_1; t) &= \\ \frac{Z^4 e^4}{2m} \int d\mathbf{v}_1 c_1^2 \frac{\partial}{\partial \mathbf{v}_1} \int dx_2 \frac{\mathbf{r}_{12} \mathbf{r}_{12}}{r_{12}^5} \cdot \frac{\mathbf{r}_{12}}{g} \left\{ \frac{\partial}{\partial \mathbf{v}_1} - \frac{\partial}{\partial \mathbf{v}_2} \right\} \int_0^1 d\lambda F^l, \end{aligned} \quad (10)$$

$F^l = g_2 f_1 f_1$  [8]. A short comment is needed for (7). First of all equation (7) is a balance equation for a kinetic part of total energy. To write the conservation law for total energy it is necessary to know also two-particle distribution function  $f_2(x_1, x_2; t)$  next to one-particle one, because the potential part of total energy is expressed via  $f_2$ . The Enskog-Landau kinetic equation in “pair collision” approximation has been obtained from the BBGKY hierarchy with a modified boundary condition in [2], where the expression for  $f_2$  is also pointed out. An average value for the potential energy and its flow one should be calculated on the basis of this expression. Then, adding it to the balance equation (7), one can obtain the conservation law for total energy.

We shall construct a normal solution to the Enskog-Landau kinetic equation (1) using the boundary conditions method [12,13]. Following this method, let us bring into the right-hand side of equation (1) an infinity small source with  $\varepsilon \rightarrow +0$ :

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v}_1 \frac{\partial}{\partial \mathbf{r}_1} \right\} f_1(x_1; t) = I_E(x_1; t) + I_{MF}(x_1; t) + I_L(x_1; t) - \varepsilon (f_1(x_1; t) - f_1^{(0)}(x_1; t)), \quad (11)$$

where  $f_1^{(0)}(x_1; t)$  is some already known one-particle distribution function satisfying equations (5) – (7) for parameters of reduced description of our system. Then the solution can be found in the form  $f_1(x_1; t) = f_1^{(0)}(x_1; t) + \delta f(x_1; t)$  and search of the normal solution implies treatment of the correction  $\delta f(x_1; t)$ . Substituting  $f_1(x_1; t)$  into (11), one can obtain:

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v}_1 \frac{\partial}{\partial \mathbf{r}_1} + \varepsilon \right\} \delta f + \frac{D}{Dt} f_1^{(0)} = I_{MF}(f_1^{(0)}) + I_{MF}(\delta f) + I_E(f_1^{(0)}, f_1^{(0)}) + I_L(f_1^{(0)}, f_1^{(0)}) + I_E(f_1^{(0)}, \delta f) + I_E(\delta f, f_1^{(0)}) + I_L(f_1^{(0)}, \delta f) + I_L(\delta f, f_1^{(0)}) + I_E(\delta f, \delta f) + I_L(\delta f, \delta f). \quad (12)$$

Conventional signs used in the equation (12) are obvious [2,8,9]. Also the fact was taken into account about  $I_{MF}(x_1; t)$ , collision integral (3), which is a functional of one-particle distribution function only. Terms with the subscript  $E$  are nonlocal, therefore in further calculations we should take their expansion with respect to the local one-particle distribution function and cut-off this expansion by terms with degrees higher than  $\delta f$ . In the case when terms with subscripts  $MF$  and  $L$  also mean nonlocal functionals, one should apply mentioned above procedure to them too. Let us combine some terms in (12):

$$\begin{aligned} I_E(\delta f) &= I_E(f_1^{(0)}, \delta f) + I_E(\delta f, f_1^{(0)}) && \text{linearized nonlocal Enskog collision functional,} \\ I_L(\delta f) &= I_L(f_1^{(0)}, \delta f) + I_L(\delta f, f_1^{(0)}) && \text{linearized Landau collision functional.} \end{aligned}$$

Now let us designate  $L_t(\delta f) = I_E^{(0)}(\delta f) + I_{MF}(\delta f) + I_L(\delta f)$  and introduce an operator  $S(t, t')$  with the following properties:

$$\frac{\partial}{\partial t} S(t, t') = L_t(\delta f) S(t, t'), \quad S(t, t')|_{t'=t} = 1.$$

Using these properties of operator  $S(t, t')$ , one can represent equation (12) in an integral form. Having correction  $\delta f(x; t)$  in an integral form, it is easy to cross to itemizing procedure for finding it in corresponding approximation. For example, it can be organized in the following way:

$$\delta f^{(k+1)}(x_1; t) = \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} S(t, t') \left\{ -\frac{D}{Dt} f_1^{(0)} - \mathbf{v}_1 \frac{\partial}{\partial \mathbf{r}_1} \delta f^{(k)} + \right. \quad (13)$$

$$\left. I_E(f_1^{(0)}, f_1^{(0)}) + I_{MF}(f_1^{(0)}) + I_L(f_1^{(0)}, f_1^{(0)}) + I_E^{(1)}(\delta f^{(k)}) \right\}_{t'},$$

where subscript  $t'$  at the bottom of right brace means that integrated expression is a function of  $t'$ . An additional condition to find  $\delta f(x; t)$  is the evident limit  $\lim_{t \rightarrow -\infty} \delta f(x; t) = 0$ . In order to construct the  $(k+1)$ -th approximation it is necessary to use the fact that  $\delta f|_{k=0} = 0$  and the conservation laws (or equations for reduced description parameters) in  $k$ -th approximation. To realize this procedure a zeroth approximation for the one-particle distribution function  $f_1^{(0)}(x_1; t)$  is needed. In the case of spherical charged particles,  $f_1^{(0)}(x_1; t)$  can be chosen as the local-equilibrium Maxwell distribution function

$$f_1^{(0)}(x_1; t) = n(\mathbf{r}_1; t) \left( \frac{m}{2\pi kT(\mathbf{r}_1; t)} \right)^{3/2} \exp \left\{ -\frac{mc_1^2(\mathbf{r}_1; t)}{2kT(\mathbf{r}_1; t)} \right\}.$$

Let us find a correction to the distribution function  $f_1^{(0)}(x_1; t)$  using itemizing procedure (13). Calculating and obtaining of conservation laws (5), (6) and equation (7), we should take into account the following relations:

$$g_2(\mathbf{r}_1, \mathbf{r}_2; t) \equiv g_2(\mathbf{r}_1, \mathbf{r}_2; n(t), \beta(t)) \rightarrow g_2(\mathbf{r}_{12}; n(t), \beta(t)), \quad (14)$$

$$F \rightarrow F^{(0)} = g_2(\mathbf{r}_{12}; n(t), \beta(t)) f_1^{(0)}(x_1; t) f_1^{(0)}(\mathbf{r}_1, \mathbf{v}_2; t),$$

where  $g_2(\mathbf{r}_{12}; n(t), \beta(t))$  is the binary quasiequilibrium correlation function, which depends on relative distance between particles. We obtain for stress tensor and heat flow vector:

$$\overset{\leftrightarrow}{P} = \overset{\leftrightarrow}{I} P^k, \quad P^k = nkT, \quad (15)$$

$$\overset{\leftrightarrow}{P} = \overset{\leftrightarrow}{I} P^{hs}, \quad P^{hs} = \frac{2}{3} \pi n^2 \sigma^3 kT g_2(\sigma|n, \beta), \quad (16)$$

$$\overset{\leftrightarrow}{P} = \overset{\leftrightarrow}{I} P^{mf}, \quad P^{mf} = -\frac{2}{3} \pi (nZe)^2 \int_{\sigma}^{\infty} \frac{dr}{r} g_2(r|n, \beta), \quad (17)$$

$$\overset{\leftrightarrow}{P}^l = 0, \quad \mathbf{q}^k = \mathbf{q}^{hs} = \mathbf{q}^{mf} = \mathbf{q}^l = 0. \quad (18)$$

In these expressions  $\overset{\leftrightarrow}{I}$  is the unit tensor,  $\overset{\leftrightarrow}{P}$  and  $\mathbf{q}^l$  are equal to zero because the integration between symmetrical limits goes over odd function. As far as calculated components  $\overset{\leftrightarrow}{P}^k(\mathbf{r}_1; t)$ ,  $\overset{\leftrightarrow}{P}^{hs}(\mathbf{r}_1; t)$ ,  $\overset{\leftrightarrow}{P}^{mf}(\mathbf{r}_1; t)$  and  $\overset{\leftrightarrow}{P}^l(\mathbf{r}_1; t)$  (15) – (18) are known, one can write total pressure in the zeroth approximation:

$$P = nkT \left( 1 + \frac{2}{3} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) - \frac{2}{3} \pi (nZe)^2 \int_{\sigma}^{\infty} \frac{dr}{r} g_2(r|n, \beta).$$

Calculating expressions in brackets on the right hand side in (13), one can write total expression for correction  $\delta f(x_1; t)$  in first approximation:

$$\begin{aligned} \delta f^{(1)}(x_1; t) = & - \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} S(t, t') \left[ f_1^{(0)}(x_1; t) \times \right. \\ & \left\{ \left( 1 + \frac{2}{5} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \left[ \frac{mc_1^2}{2kT} - \frac{5}{2} \right] \mathbf{c}_1 \frac{\partial}{\partial \mathbf{r}_1} \ln T(\mathbf{r}_1; t) + \right. \\ & \left. \left( 1 + \frac{4}{15} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \frac{m}{kT} \left[ \mathbf{c}_1 \mathbf{c}_1 - \frac{1}{3} c_1^2 \overset{\leftrightarrow}{I} \right] : \frac{\partial}{\partial \mathbf{r}_1} \mathbf{V}(\mathbf{r}_1; t) \right\} \Bigg]_{t'}. \end{aligned} \quad (19)$$

Terms related to short-range interactions only contribute evidently into the correction in this approximation. Contrary to the kinetic theory of dilute gases particle sizes take part here [6,7,11], where particles are considered as point-like objects. Nevertheless, the influence of both long-range and short-range parts of interactions are also “hidden” in operator  $S(t, t')$  (through operator  $L_t$ ). Formally, the expression (19) looks completely the same as the correction in [18]. But a difference lies in the structure of the operator  $S(t, t')$ .

Having total expression for correction  $\delta f(x_1; t)$  in the first approximation (19) one can calculate conservation laws (5), (6) and equation (7) in the same approximation. Therefore, it is necessary, first, to obtain relations for determining quantities (7) in which the correction (19) can be engaged. For  $\overset{\leftrightarrow}{P}^{k1}(\mathbf{r}_1; t)$  we obtain:

$$\overset{\leftrightarrow}{P}^{k1}(\mathbf{r}_1; t) = \overset{\leftrightarrow}{I} P^k - \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} M^k(t, t') \left[ \overset{\leftrightarrow}{S} \right]_{t'}, \quad (20)$$

where  $S_{\alpha\beta}$  is a velocity shift tensor,

$$M^k(t, t') = \frac{m}{5} \int d\mathbf{v}_1 \mathbf{c}_1 \mathbf{c}_1 S(t, t') \times \quad (21)$$

$$\left[ f_1^{(0)}(x_1; t) \left( 1 + \frac{4}{15} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \frac{m}{kT} \left( \mathbf{c}_1 \mathbf{c}_1 - \frac{1}{3} c_1^2 \vec{I} \right) \right]_{t'}$$

is a kernel of kinetic part of the transport equations.

For calculating  $\overleftrightarrow{P}^{hs\ 1}(\mathbf{r}_1; t)$ ,  $\overleftrightarrow{P}^{mf\ 1}(\mathbf{r}_1; t)$  and  $\overleftrightarrow{P}^{l\ 1}(\mathbf{r}_1; t)$ , we have to expand  $F^{hs}$ ,  $F^{mf}$  and  $F^l$  on inhomogeneity and deviation  $\delta f(x_1; t)$  and keep in the series initial terms only. The expansion for  $F^{mf}$ ,  $F^l$  reads the same as for  $F^{hs}$  with changing  $g_2(\sigma|n, \beta) \rightarrow g_2(\mathbf{r}_{12}|n, \beta)$ ,  $\hat{\mathbf{r}}_{12} \rightarrow |\mathbf{r}_{12}|^{-1} \mathbf{r}_{12}$ ,  $\sigma \rightarrow |\mathbf{r}_{12}|$ . The calculations show:

$$\overleftrightarrow{P}^{hs\ 1}(\mathbf{r}_1; t) = \overleftrightarrow{P}^{hs} - \frac{4}{9} n^2 \sigma^4 g_2(\sigma|n, \beta) \sqrt{\pi m k T} \left[ \frac{6}{5} \vec{S} + (\nabla \cdot \mathbf{V}) \vec{I} \right] - \quad (22)$$

$$\frac{4}{15} \pi n \sigma^3 g_2(\sigma|n, \beta) \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} M^k(t, t') \left[ \vec{S} \right]_{t'},$$

$$\overleftrightarrow{P}^{mf\ 1}(\mathbf{r}_1; t) = \vec{I} P^{mf}. \quad (23)$$

A mean field influence into the total stress tensor remains the same as in zeroth approximation. Similar situation arises as to  $\overleftrightarrow{P}^{l\ 1}(\mathbf{r}_1; t)$ :

$$\overleftrightarrow{P}^{l\ 1}(\mathbf{r}_1; t) = \overleftrightarrow{P}^l(\mathbf{r}_1; t) = 0. \quad (24)$$

Total expression for stress tensor in the first approximation is a sum of (20), (22), (23) and (24):

$$\begin{aligned} \vec{P}(\mathbf{r}_1; t) = & \vec{I} P(\mathbf{r}_1; t) - \frac{4}{9} n^2 \sigma^4 g_2(\sigma|n, \beta) \sqrt{\pi m k T} \left[ \frac{6}{5} \vec{S} + (\nabla \cdot \mathbf{V}) \vec{I} \right] - \\ & \left( 1 + \frac{4}{15} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} M^k(t, t') \left[ \vec{S} \right]_{t'}. \end{aligned}$$

The calculations for heat flow vectors give:

$$\mathbf{q}^{k\ 1}(\mathbf{r}_1; t) = - \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} L^k(t, t') \left[ \frac{1}{T} \nabla T \right]_{t'}, \quad (25)$$

$$\mathbf{q}^{hs\ 1}(\mathbf{r}_1; t) = - \frac{2}{3} n^2 \sigma^4 g_2(\sigma|n, \beta) \sqrt{\frac{\pi k^3 T}{m}} \nabla T(\mathbf{r}_1; t) - \quad (26)$$

$$\frac{2}{5} \pi n \sigma^3 g_2(\sigma|n, \beta) \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} L^k(t, t') \left[ \frac{1}{T} \nabla T \right]_{t'},$$



$$\mathbf{q}^{l1}(\mathbf{r}_1; t) = \mathbf{q}^l(\mathbf{r}_1; t) = 0. \quad (27)$$

Here

$$L^k(t, t') = \frac{1}{3} \int d\mathbf{v}_1 c_1 \frac{mc_1^2}{2} S(t, t') \times \left[ f_1^{(0)}(x_1; t) \left( 1 + \frac{2}{5} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \left( \frac{mc_1^2}{2kT} - \frac{5}{2} \right) c_1 \right]_{t'} \quad (28)$$

is another kernel of kinetic part of transport equations. Total expression for heat flux vector is a sum of (25) – (27):

$$\mathbf{q}(\mathbf{r}_1; t) = -\frac{2}{3} n^2 \sigma^4 g_2(\sigma|n, \beta) \sqrt{\frac{\pi k^3 T}{m}} \nabla T(\mathbf{r}_1; t) - \left( 1 + \frac{2}{5} \pi n \sigma^3 g_2(\sigma|n, \beta) \right) \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} L^k(t, t') \left[ \frac{1}{T} \nabla T \right]_{t'}.$$

Now we can consider one of the limiting cases, namely, the stationary process, when the operator  $L_t$  does not depend on time, i.e.  $S(t, t') = \exp \{L_t(t - t')\}$ . Some terms in expressions for  $\vec{P}(\mathbf{r}_1; t)$  and  $\mathbf{q}(\mathbf{r}_1; t)$  can acquire simpler form. We can compare them with those from the Enskog-Landau kinetic equation for one-component system of charged hard spheres with using the Chapman-Enskog method in the case, when in a long-range part of the collision integral we put  $g_2(\sigma|n, \beta) \rightarrow 1$ . It should be noted that bulk viscosity has the same structure as in the Chapman-Enskog method [8]. But other transport coefficients exhibit some distinctions. The structure for shear viscosity  $\eta$  and thermal conductivity  $\lambda$  is:

$$\eta = \frac{3}{5} \varpi + 2 nkT \frac{\left\{ 1 + \frac{4}{15} \pi n \sigma^3 g_2(\sigma|n, \beta) \right\}^2}{\left\{ I_E^{(0)}(\delta f) + I_L(\delta f) \right\}}, \quad (29)$$

$$\lambda = \frac{3k}{2m} \varpi + \frac{5k}{m} nkT \frac{\left\{ 1 + \frac{2}{5} \pi n \sigma^3 g_2(\sigma|n, \beta) \right\}^2}{\left\{ I_E^{(0)}(\delta f) + I_L(\delta f) \right\}}. \quad (30)$$

Then the problem lies in calculating collision integrals  $I_E^{(0)}(\delta f)$  and  $I_L(\delta f)$ , this means that we should calculate collision integrals (2) (in the zeroth approximation on inhomogeneity) and (4) together in the first approximation

on deviation  $\delta f$ , where  $\delta f$  is substituted from (19). The matter of some difficulty is that correction (19) in its turn is expressed also via collision integrals  $I_E^{(0)}(\delta f)$ ,  $I_L(\delta f)$ , which are in the operator  $S(t, t')$ . So the first acceptable approximation should be that, when correction  $\delta f$  (19) is expressed via  $I_E^{(0)}(\delta f)$ ,  $I_L(\delta f)$  calculated with  $\delta f'$ , where  $\delta f' = \delta f$  at  $S(t, t') = 1$ . For  $I_E^{(0)}(\delta f)$  we obtain the results [8], for  $I_L(\delta f)$  in (29), (30) we can obtain the following:

$$I_L(\delta f) = \frac{Z^4 e^4}{m^2} \frac{\partial}{\partial \mathbf{v}_1} \int d\mathbf{r}_{12} d\mathbf{v}_2 g_2(\mathbf{r}_1, \mathbf{r}_1 + \mathbf{r}_{12}; t) \frac{\mathbf{r}_{12} \mathbf{r}_{12}}{r_{12}^5} \frac{1}{g} \left\{ \frac{\partial}{\partial \mathbf{v}_1} - \frac{\partial}{\partial \mathbf{v}_2} \right\} \\ \times \left\{ f_1(x_1; t) \delta f(\mathbf{r}_1 + \mathbf{r}_{12}, \mathbf{v}_2; t) + \delta f(x_1; t) f_1(\mathbf{r}_1 + \mathbf{r}_{12}, \mathbf{v}_2; t) \right\}, \quad (31)$$

where  $\delta f(x; t)$  is evaluated from (19) with

$$S(t, t') = \exp \left\{ L_t(t - t') \right\} = \exp \left\{ [I_E^{(0)}(\delta f') + I_L(\delta f')](t - t') \right\}$$

at  $\delta f'(x; t) = \delta f(x; t) \big|_{S(t, t') = 1}$ . This stage of calculations needs an explicit form for the binary quasiequilibrium correlation function  $g_2$  both on the contact and in  $\mathbf{r}$ -space.

The results of this paper (19), (29) and (30) will coincide completely with those from [8] when in a long-range part of the collision integral (31) one puts  $g_2(\mathbf{r}_1, \mathbf{r}_1 + \mathbf{r}_{12}) \equiv 1$  and represents it in Boltzmann-like form. But the used boundary conditions method has turned out more convenient than the Chapman-Enskog one [2,8]. As was discussed in details in [12] at constructing the normal solution for a kinetic equation using the boundary conditions method, time derivatives  $\partial/\partial t$  of hydrodynamic parameters of reduced description do not set to be small. Therefore, the normal solution to this equation could be used for hydrodynamic description of fast processes.

## References

- [1] D.N.Zubarev, V.G.Morozov, Teor. Mat. Fiz. 60(1984)270 (in Russian).
- [2] D.N.Zubarev, V.G.Morozov, I.P.Omelyan, M.V.Tokarchuk, Teor. Mat. Fiz. 87(1991)113 (in Russian).
- [3] D.N.Zubarev, V.G.Morozov, I.P.Omelyan, M.V.Tokarchuk, Teor. Mat. Fiz. 96(1993)325 (in Russian).
- [4] Yu.L.Klimontovich, Teor. Mat. Fiz. 92(1992)312 (in Russian).
- [5] Yu.L.Klimontovich, Phys. Lett. A 170(1992)434.

- [6] N.N.Bogoliubov, Problems of a dynamical theory in statistical physics, in: Studies in statistical mechanics, vol. 1 (eds. J. de Boer and G.E.Uhlenbeck) (North-Holland, Amsterdam, 1962).
- [7] D.N.Zubarev, M.Yu.Novikov, Teor. Mat. Fiz. 13(1972)406 (in Russian).
- [8] A.E.Kobryn, V.G.Morozov, I.P.Omelyan, M.V.Tokarchuk, Physica A 230 (1996)189.
- [9] A.E.Kobryn, I.P.Omelyan, Enskog-Landau kinetic equation for two-component dense plasma. The solution, transport coefficients, in: Proceedings, Contributed papers of International Conference "Physics in Ukraine", volume: Plasma physics, (Kiev, June 22-27, 1993), p. 135-138.
- [10] H.Grad, Commun. Pure & Appl. Math. 2(1949)311.
- [11] V.M.Zhdanov, Transport phenomena in multi-component plasma (Energoatomizdat, Moscow, 1982) (in Russian).
- [12] D.N.Zubarev, A.D.Khonkin, Teor. Mat. Fiz. 11(1972)403 (in Russian).
- [13] G.O.Balabanian, A.D.Khonkin, Teor. Mat. Fiz. 18(1974)130 (in Russian).
- [14] M.H.Ernst, H. van Beijeren, Physica 68(1973)437.
- [15] J.Karkheck, G.Stell, J. Chem. Phys. 75(1981)1475.
- [16] G.Stell, J.Karkheck, H. van Beijeren, J. Chem. Phys. 79(1983)3166.
- [17] D.N.Zubarev, Nonequilibrium statistical thermodynamics (Consultant Bureau, New York, 1974).
- [18] M.V.Tokarchuk, I.P.Omelyan, Normal solution of Enskog-Vlasov kinetic equation using boundary conditions method, in: Proceedings, Contributed papers of conference "Modern Problems of Statistical Physics", vol. 1, (Lviv, February 3-5, 1987), p. 245-252 (in Russian).